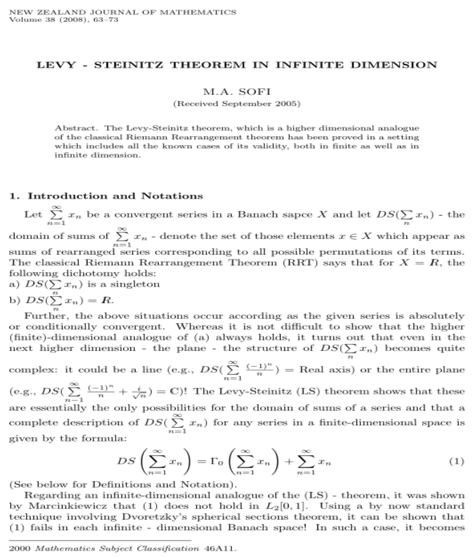
Rearrangements Of Series In Banach Spaces



NEW ZEALAND JOURNAL OF MATHEMATICS
Volume 38 (2008), 63–73

LEVY - STEINITZ THEOREM IN INFINITE DIMENSION

M.A. SOFI

(Received September 2005)

Abstract. The Levy-Steinitz theorem, which is a higher dimensional analogue of the classical Riemann Rearrangement theorem has been proved in a setting which includes all the known cases of its validity, both in finite as well as in infinite dimension.

1. Introduction and Notations

Let $\sum_{n=1}^{\infty} x_n$ be a convergent series in a Banach sapce X and let $DS(\sum_n x_n)$ - the domain of sums of $\sum_{n=1}^{\infty} x_n$ - denote the set of those elements $x \in X$ which appear as sums of rearranged series corresponding to all possible permutations of its terms. The classical Riemann Rearrangement Theorem (RRT) says that for $X = R$, the following dichotomy holds:
a) $DS(\sum_n x_n)$ is a singleton
b) $DS(\sum_n x_n) = R$.

Further, the above situations occur according as the given series is absolutely or conditionally convergent. Whereas it is not difficult to show that the higher (finite)-dimensional analogue of (a) always holds, it turns out that even in the next higher dimension - the plane - the structure of $DS(\sum_n x_n)$ becomes quite complex: it could be a line (e.g., $DS(\sum_{n=1}^{\infty} \frac{(-1)^n}{n})$ = Real axis) or the entire plane (e.g., $DS(\sum_{n-1}^{\infty} \frac{(-1)^n}{n} + \frac{i}{\sqrt{n}}) = \mathbb{C}$)! The Levy-Steinitz (LS) theorem shows that these are essentially the only possibilities for the domain of sums of a series and that a complete description of $DS(\sum_{n=1}^{\infty} x_n)$ for any series in a finite-dimensional space is given by the formula:

$$DS\left(\sum_{n=1}^{\infty} x_n\right) = \Gamma_0\left(\sum_{n=1}^{\infty} x_n\right) + \sum_{n=1}^{\infty} x_n \quad (1)$$

(See below for Definitions and Notation).

Regarding an infinite-dimensional analogue of the (LS) - theorem, it was shown by Marcinkiewicz that (1) does not hold in $L_2[0,1]$. Using a by now standard technique involving Dvoretzky's spherical sections theorem, it can be shown that (1) fails in each infinite - dimensional Banach space! In such a case, it becomes

2000 *Mathematics Subject Classification* 46A11.

In a contemporary course in mathematical analysis, the concept of series arises as a natural generalization of the concept of a sum over finitely.Rearrangements of series in Banach spaces. A short survey. Vladimir Kadets. Department of Mechanics and Mathematics. Kharkiv redaalc.comn National.for any rearrangement of its terms, but does not converge absolutely since Series in Banach Spaces: Conditional and Unconditional.S. A. Chobanyan The structure of the set of sums of a conditionally convergent series in a normed space Mat. Sb. () S. A. Chobanyan REARRANGEMENTS OF SERIES IN BANACH. SPACES AND ARRANGEMENTS OF SIGNS. To cite this article: D V Pecherski Math. USSR Sb. 63 This article is cited in 4 scientific papers (total in 4 papers) Rearrangements of series in Banach spaces and arrangements of signs D. V. Pecherskii Abstract.Conditional and Unconditional Convergence Vladimir Kadets. 3. Preliminary Material on Rearrangements of Series of Elements of a Banach Space From this .Title, Rearrangements of Series in Banach Spaces. Author, Vladimir M. Kadet?s?. Publisher, American Mathematical Soc. ISBN, , This paper is devoted to some applications of probability theory in infinite dimensional spaces to problems of analysis related to rearrangements of summands in.In this case the theory of series is a part of the theory of sequences, whose terms are elements of Banach (as well as other topological linear) spaces. when a series converges only for certain rearrangements of its terms (in other words.k=1 a?(k) for some ?}. It is the set of sums of all the rearrangements of the series. .. Every Banach space E of infinite dimension contains a series whose set of.This is indeed a rearrangement of the alternating harmonic series: every odd integer occurs once positively, and . Rearrangements of series in Banach spaces.A series converges unconditionally if every rearrangement of the series converges in R. An absolutely convergent series in a Banach space.download rearrangements of series in banach spaces representatives imagine entitled in the ground-breaking to add its object under the.Let X be a real Banach space and ?. X be its topological dual. We remind that a series ?k k a in X converges unconditionally if each of its rearrangements ?k.In mathematics, an infinite series of numbers is said to converge absolutely if the sum of the For instance, rearrangements do not change the value of the sum. .. Since a series with values in a finite-dimensional normed space is absolutely convergent if each of its one-dimensional projections is absolutely convergent.Garsia, A. (). Rearrangements for Fourier series. Ann. Math. 79, Garsia, A. (). Topics in Almost Everywhere Convergence. Markham, Chicago.